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Return To Recurrent Formula In Calculating Definite Integrals

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Annotation: briefly introducing the content of the article, this article covers another convenient way of calculating exact integrals. That is, if the calculation of the exact integral of the sequence is given, then it will be appropriate for us to use recurrent formulas. How is a recurring formula created? You can find the answer to this question by reading the article.
Keywords: integral, definite integral, recurrent sequence.

Aniq Integrallarni Hisoblashda Rekkurent Formulaga Keltirish

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Anotatsiya: maqola mazmunini qisqacha tanishtiradigan bo`lsak, ushbu maqola aniq integrallarni hisoblashning yana bir qulay usulini yoritadi. Ya'ni ketma – ketlikning aniq integralini hisoblash berilgan bo`lsa, u holda biz rekkurent formulalardan foydalanishimiz o`rinli bo`ladi. Rekkurent formula qanday hosil qilinadi? Ushbu savolga javobni maqolani o`qib olishingiz mumkin.
Kalit so`zlar: integral, aniq integral, rekkurent ketma-ketlik.

As it is known from analysis courses, if all subsequent terms of a sequence starting from one term can be expressed by the preceding terms, such sequences are called recurring or recurrent sequences. The Greek meaning of the word recurrent is derived from the word reccurrere - to return. The most important thing in calculating definite integrals is to determine this recurrent sequence. Such recurrent formulas are widely used in several theoretical and practical fields of analysis. We will consider the reduction of some definite integrals to the recurrent formula.

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Example 1. $n \in N$, $I_n = \int_0^1 (1-x^2)^n dx$

In calculating this integral, we use integration by pieces, i.e

$u = (1-x^2)^n, du = (n-1)(1-x^2)^{n-1}(-2x), dv = dx, v = x$.

As a result, we have the following expression:

$$I_n = (1-x^2)^n x\Big|_0^1 + 2(n-1)\int_0^1 (1-x^2)^{n-1} x^2 dx =$$

$$= (1-x^2)^n x\Big|_0^1 - 2(n-1)\int_0^1 (1-x^2)^{n-1}(1-x^2)dx + 2(n-1)\int_0^1 (1-x^2)^{n-1} dx$$

$$I_n = 2(n-1)x\Big|_0^1 - 2(n-1)I_n + 2(n-1)I_{n-1}$$

Here uv Resetting the limits to 2 turns this expression into 0. So, $I_n = \frac{2n}{1+2n} I_{n-1}$ the recurring formula is derived.

On this I_n, in sequence I_1 or I_0 we can express with

$$I_0 = 1, I_1 = \frac{2}{3}, I_2 = \frac{4}{5}\cdot\frac{2}{3}, I_3 = \frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}, I_4 = \frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}.$$

These are the conclusions $I_n = \frac{(2n)!!}{(2n+1)!!}$ we can take that.

EXAMPLE 2. Calculate the following integral when n is natural:

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx, I_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$$

Integrating by pieces, that is

$u = \sin^{n-1} x, du = (n-1)\sin^{n-2}, dv = d(-\cos x), v = -\cos x$

$$I_n = \int_0^{\frac{\pi}{2}} \sin^{n-1} d(-\cos x) = -\sin^{n-1}\cos x\Big|_0^{\frac{\pi}{2}} + (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x dx$$

we generate .uv Resetting the bounds to 2 makes it zero. $\cos^2 x = 1 - \sin^2 x$ performing the trigonometric substitution, we get the following integral:

$$I_n = (n-1)I_{n-2} - (n-1)I_n$$

From this

$$I_n = \frac{n-1}{n} I_{n-2}$$

A recurring formula is derived. On this I_n consecutively I_0 or I_1 a general formula can be determined by finding For example, $n = 2m$ when

$$I_{2m} = \int_0^{\frac{\pi}{2}} \sin^{2m} x dx = \frac{(2m-1)(2m-3)\cdot \ldots \cdot 3\cdot 1}{2m(2m-2)\cdot \ldots \cdot 4\cdot 2}\cdot\frac{\pi}{2}$$

if m=2n+1

$$I_{2m+1} = \int_0^{\frac{\pi}{2}} \sin^{2m+1} x dx = \frac{2m(2m-2)\cdot \ldots \cdot 4\cdot 2}{(2m+1)(2m+1)\cdot \ldots \cdot 3\cdot 1}$$

will be. In order to write the found expressions shorter $m!!$we enter the symbol. It means the product of numbers that are not a multiple of and are in the same pair as it (for example).m $7!! = 1\cdot 3\cdot 5\cdot 7$

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx = \frac{(n-1)!!}{n!!}\frac{\pi}{2}, \quad agar\ n-juft$$ if

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx = \frac{(n-1)!!}{n!!}, \quad agar\ n-toq$$ if

$$I_n = \int_0^{\frac{\pi}{2}} \cos^n dx$$ For integral $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$ using the trigonometric substitution, we perform the same operations as the first integral.

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