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Teaching Of The Extrema Of The Function In Mathematical Lessons

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Abstract: in this article discusses about teaching of the extrema of the function in mathematical lessons.
Key words: extrema, function, mathematical, lesson.

Let us consider a function $y = f(x)$defined on the interval $(a, b)$of the numerical axis R. Let the point be $x_0 \in (a, b)$.

Definition 1. The interval $(x_0 - \delta, x_0 + \delta)$, where $\delta > 0$is called δ*the neighborhood of the point* $x_0$and is denoted by $U_\delta(x_0)$:

$$U_\delta(x_0) = \{x \in R:\ x_0 - \delta < x < x_0 + \delta;\ \delta > 0\}.$$

Definition 2. If exists δ, then the neighborhood of the point x_0 $U_\delta(x_0) \subset (a, b)$such that for any $x \in U_\delta(x_0)$equality holds

$$f(x) \le f(x_0),$$

that's what it x_0's called *point of local maximum* of the function $f(x)$.

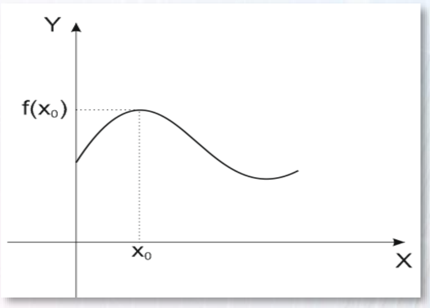

Fig. 1a.

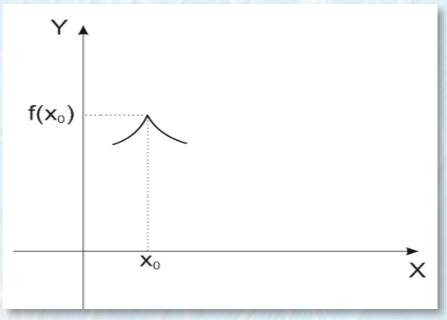

Fig. 1b.

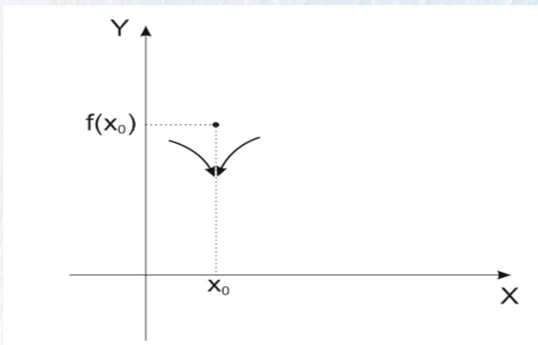

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Fig. 1c.

Definition 3. If exists δ, then the neighborhood of the point x_0 $U_\delta(x_0) \subset (a, b)$such that for any $x \in U_\delta(x_0)$equality holds

$$f(x) \geq f(x_0),$$

that's what it x_0's called *point of local minimum* of the function $f(x)$.

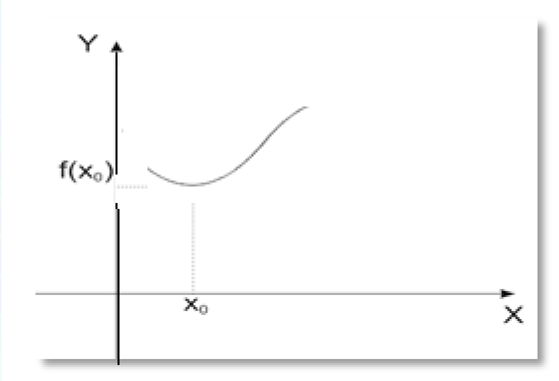
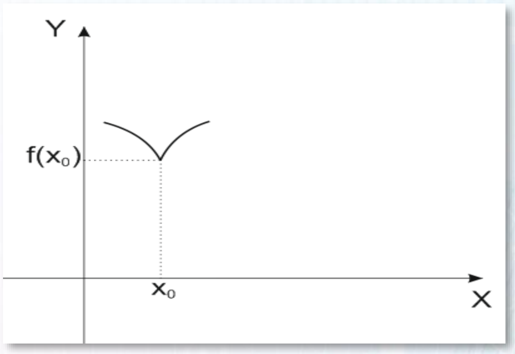


Fig. 2a.



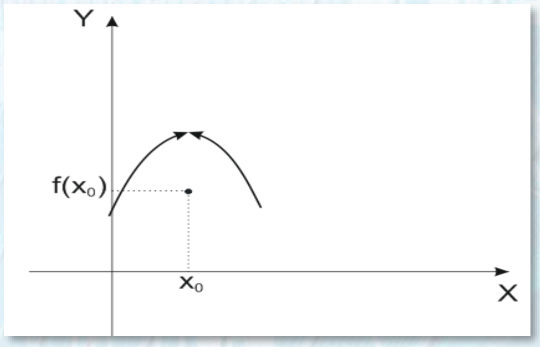


Fig. 2b.

Fig. 2c.

In the future, for brevity, we will call the points of the local maximum of the function $f(x)$the points of the maximum of the function, and $f(x_0)$we will denote the values of the maximum of the function by$f(x)$

$$f(x_0) = \max_{x \in U_\delta(x_0)} f(x).$$

Similarly, we will call the points of the local minimum of the function $f(x)$the points of the minimum of the function, and $f(x_0)$we will denote the values by the minimum of the function$f(x)$

$$f(x_0) = \min_{x \in U_\delta(x_0)} f(x).$$

Figures 1 a) – 1 c) show maximum points, and figures show minimum points.

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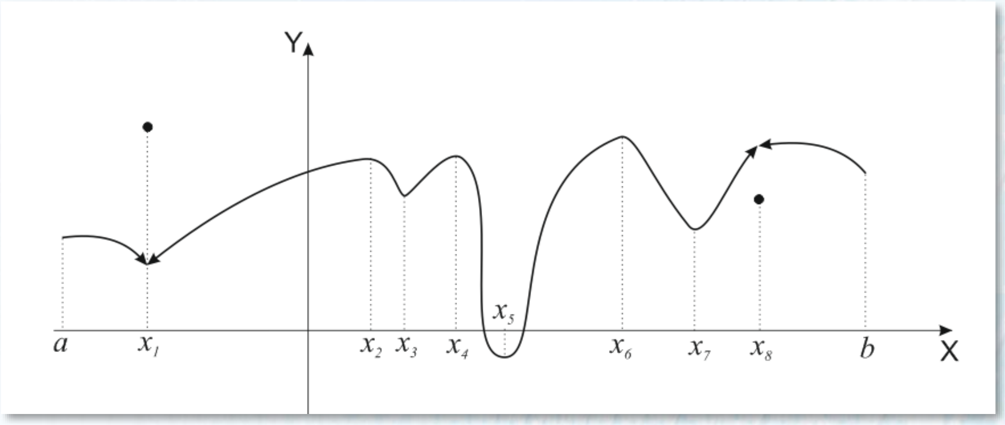
The points of maximum and minimum of a function are called *extreme points,* and the maximum and minimum of a function are called *extrema* (extreme values of a function).

Example 1. Let us consider a function defined on an interval (a, b)graphically :



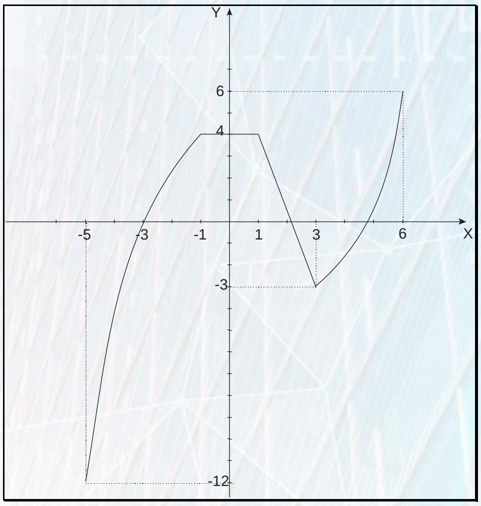
According to definitions 2–3, points $x_1, x_2, x_4, x_6$are maximum points, and points $x_3, x_5, x_7, x_8$are minimum points of the given function.

Example 2. Let's consider the function

$$f(x) = \begin{cases} -x^2 - 2x + 3, & -5 < x \leq -1 \\ 4, & -1 < x \leq 1 \\ \dfrac{15 - 7x}{2}, & 1 \leq x < 3 \\ x^2 - 6x + 6, & 3 < x < 6 \end{cases}$$

on the interval $(-5,6)$.

The graph of this function has the form (see example 2. from § 1):



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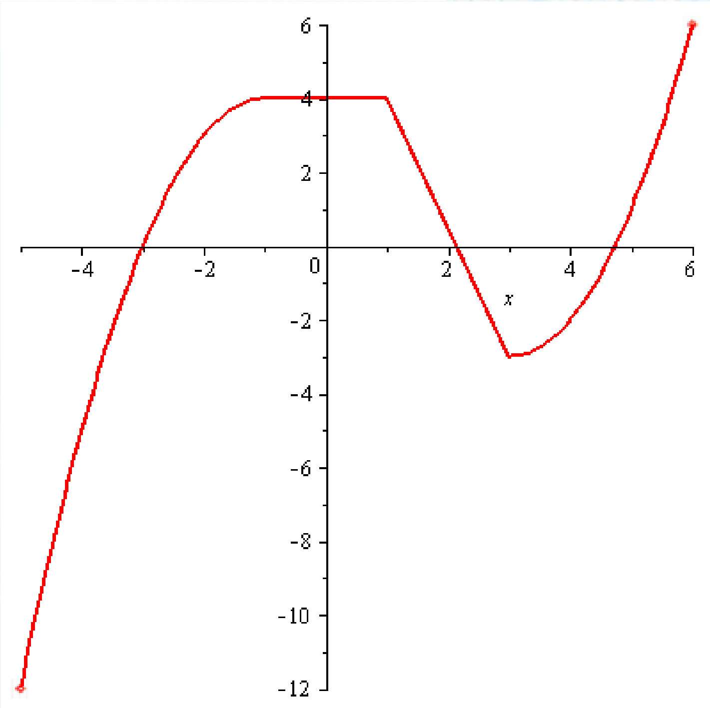
The point $x_1 = 3$is a minimum point of the function $f(x)$, according to definition 3, and any point on the segment is $[-1,1]$a maximum point, according to definition 2.

In maple

$$h := x \rightarrow piecewise\left(-5 \le x \textbf{ and } x \le -1, -x^2 - 2\cdot x + 3, -1 < x \textbf{ and } x \le 1, 4, 1 < x \textbf{ and } x < 3, \frac{(15 - 7\cdot x)}{2}, 3 \le x \textbf{ and } x \le 6, x^2 - 6\cdot x + 6\right)$$

$$h := x \rightarrow piecewise\left(-5 \le x \textbf{ and } x \le -1, -x^2 - 2x + 3, -1 < x \textbf{ and } x \le 1, 4, 1 < x \textbf{ and } x < 3, \frac{15}{2} - \frac{7}{2}x, 3 \le x \textbf{ and } x \le 6, x^2 - 6x + 6\right)$$

>plot(h(x), x = -5 ..6, color = red, thickness = 2, discont = true)



Definition 4. The point at which the derivative of a function $f(x)$vanishes or does not exist is called *a critical* point $f(x)$.

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Theorem 1. (*Necessary condition for an extremum)* . If a point $x_0 \in (a,b)$is an extremum point of a function $f(x)$, then $x_0$is a critical point of the function $f(x)$.

This means that, based on Theorem 1, the extremum points of a function $f(x)$should be sought among its critical points.

The question "will the function reach its extremum at these points or not" can be answered using sufficient conditions for an extremum. To form them, we introduce the concepts of left and right neighborhoods of a point $x_0 \in (a,b)$.

Definition 5. The interval $(x_0-\delta, x_0) \subset (a,b)$, where $\delta > 0$, is called *the left δneighborhood of the point.* $x_0$and is denoted by $U_\delta^-(x_0)$:

$$U_\delta^-(x_0) = \{x \in R:\ x_0-\delta < x < x_0;\ \delta > 0\}.$$

Definition 6. The interval $(x_0, x_0+\delta) \subset (a,b)$, where $\delta > 0$, is called *the right δ-hand neighborhood of the point.* $x_0$and is denoted by $U_\delta^+(x_0)$:

$$U_\delta^+(x_0) = \{x \in R:\ x_0 < x < x_0+\delta;\ \delta > 0\}.$$

Theorem 2. (*First-order sufficient condition).* Let the function $f(x)$be continuous at a point $x_0$and have a finite derivative on $U_\delta(x_0)\backslash\{x_0\}$.

a) if for any $x \in U_\delta^-(x_0)$ $f'(x) > 0$and for any
$x \in U_\delta^+(x_0)$ $f'(x) < 0$, then $x_0$it is the maximum point of the function $f(x)$;

b) if for any $x \in U_\delta^-(x_0)$ $f'(x) < 0$and for any
$x \in U_\delta^+(x_0)$ $f'(x) > 0$, then $x_0$it is the minimum point of the function $f(x)$;

c) if for any $x \in U_\delta^-(x_0)$ $f'(x) > 0$and for any
$x \in U_\delta^+(x_0)$ $f'(x) > 0$, or for any $x \in U_\delta^-(x_0)$ $f'(x) < 0$and for any $x \in U_\delta^+(x_0)$ $f'(x) < 0$, then $x_0$it is not an extreme point of the function $f(x)$.

In other words, if the derivative of a function $f(x)$changes sign when passing through a critical point x_0, then the function $f(x)$has an extremum at this point. If the derivative of a function $f(x)$does not change sign when passing through a critical point x_0, then the function does not have an extremum at this point $f(x)$.

Example 3. Investigate the function for extremum

$$f(x) = \frac{x^2}{x^2+3}.$$

Solution. The domain of the function $f(x)$is the entire number line R. According to Theorem 1, we find the critical points of the function $f(x)$, for

which, according to the rule for finding the derivative of a ratio, we obtain the derivative
$f'(x)$this function

$$f'(x) = \frac{2x(x^2+3) - 2x^2x}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2}.$$

Using the definition of a critical point of a function, $f(x)$first equating $f'(x)$to zero, i.e.

$$\frac{6x}{(x^2+3)^2} = 0,$$

we determine that the last equality holds for $x = x_0 = 0$. Since the function $f(x)$has finite derivatives in R, the only critical point of the function $f(x)$is x_0.

Let us determine the signs of the derivative of the function $f(x)$in the left and right neighborhoods of the point x_0:

for anyone$x \in U_\delta^-(0) = \{x \in R: -\delta < x < 0;\ 0 < \delta < 1\}$

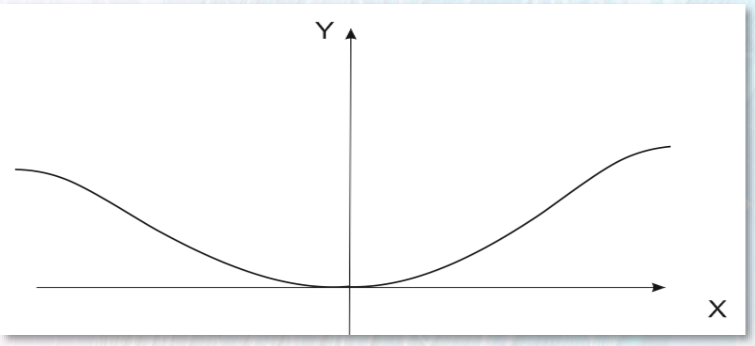
$$f'(x) = \frac{6x}{(x^2+3)^2} < 0, "\ -\ ",$$

for anyone$x \in U_\delta^+(0) = \{x \in R: 0 < x < \delta;\ 0 < \delta < 1\}$

$$f'(x) = \frac{6x}{(x^2+3)^2} > 0,\ +.$$

This means that the derivative of the given function changes sign from minus ("- ") to plus ("+") when passing through the critical point x_0. The given function is continuous at the point x_0. Therefore, according to Theorem 2, $x_0$is the minimum point of the given function and its minimum is equal to

$$f(x_0) = \min_{x \in U_\delta(x_0)} f(x).$$



In maple

$>f := \frac{x^2}{x^2+3};$

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$$f := \frac{x^2}{x^2+3}$$

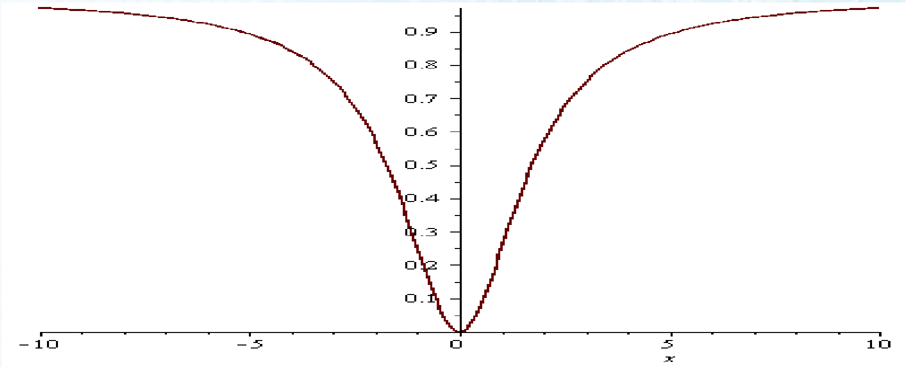
> $\frac{d}{dx} f;$

$$\frac{2x}{x^2+3} - \frac{2x^3}{(x^2+3)^2}$$

> $solve\left(\frac{2x}{x^2+3} - \frac{2x^3}{(x^2+3)^2}, x\right);$

$$0$$

> $plot(f);$



Example 4. Investigate the function for extremum

$$f(x) = x^{\frac{2}{3}}.$$

In maple

> $\sqrt[3]{x^2};$

$$(x^2)^{1/3}$$

> smartplot ( );

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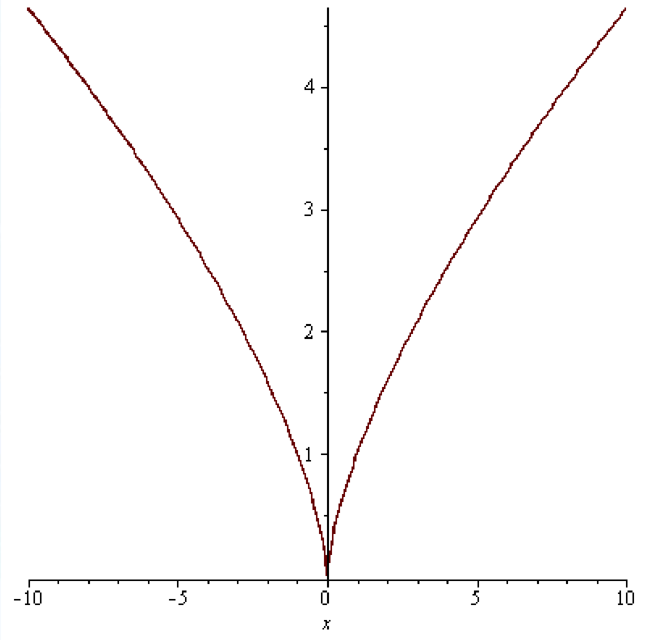

Solution : The given function is defined and continuous on the entire number axis. The derivative of the given function, when $x \neq 0$ is defined as:

$$f'(x) = \frac{2}{3\sqrt[3]{x}}.$$

> diff ((x^ 2)^ (1/3), x);

$$\frac{2}{3} \frac{x}{\left(x^2\right)^{2/3}}$$

Derivative of a function $f(x)$ at $x = 0$ does not exist:

$$\lim_{\Delta x \to +0} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to +0} \frac{1}{\Delta x^{\frac{1}{3}}} = +\infty;$$

$$\lim_{\Delta x \to -0} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to -0} \frac{1}{\Delta x^{\frac{1}{3}}} = -\infty.$$

(the function $f'(x)$ suffers a discontinuity of the second kind (an infinite jump at the point $x = 0$)).

This means that the point $x = 0$ is a critical point of the function $f(x)$ according to Definition 4. When passing through the critical point, $x = 0$ the derivative $f'(x)$ changes sign from minus ("-") to plus ("+"). The given function

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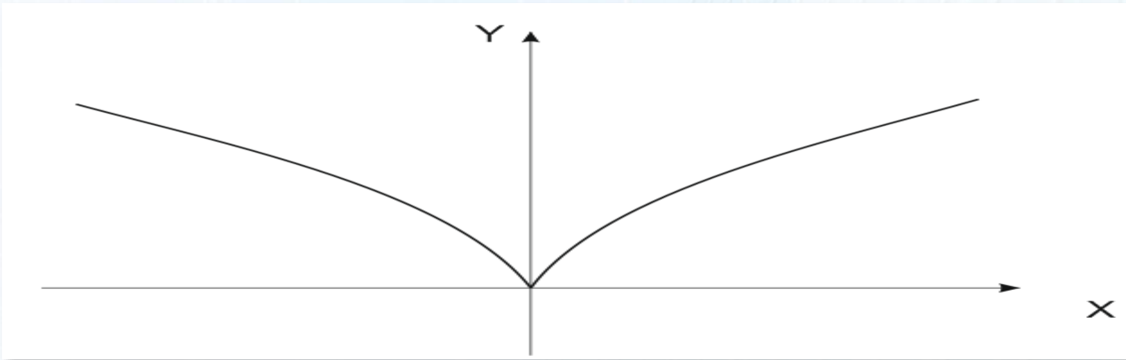
is continuous at the critical point. According to Theorem 2, the critical point $x = 0$is the minimum point of the function $f(x)$and its minimum is equal to

$$f(0) = \min_{x \in U_\delta(0)} f(x) = 0.$$



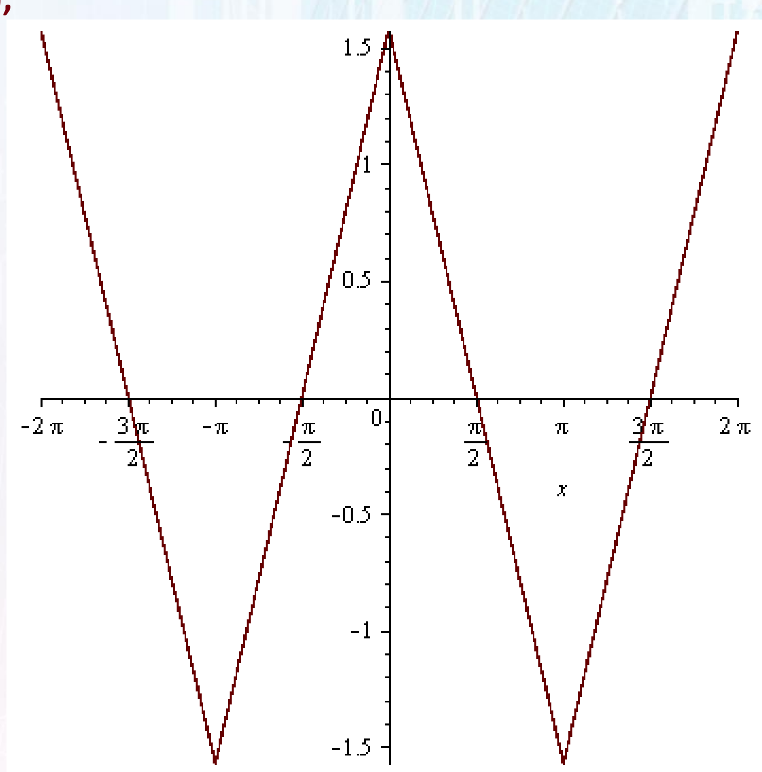
Example 5. Investigate the function for extremum

$$f(x) = arcsin(cosx).$$

>arcsin(cos(x))

$$\frac{1}{2}\pi - \arccos(\cos(x))$$

> smartplot ( );



Solution . The domain of the function $f(x)$is the entire number line. Let us find the expression for the derivative of this function as a complex function:

$$f'(x) = \frac{(cosx)'}{\sqrt{1-cos^2x}} \cdot \frac{-sinx}{\sqrt{1-cos^2x}} = -\frac{sinx}{|sinx|}.$$

> diff (, x);

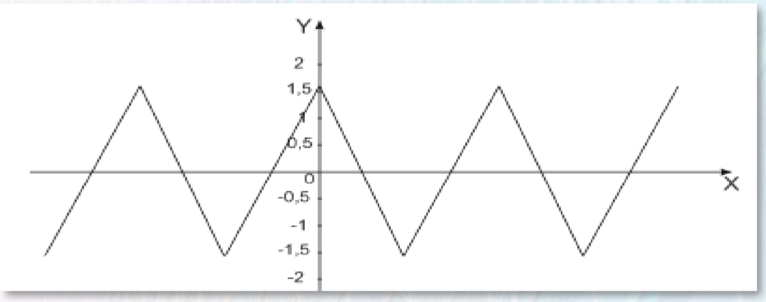
$$-\frac{\sin(x)}{\sqrt{1-\cos(x)^2}}$$

From the form of the expression for the derivative of the function $f(x)$, we conclude that the given function has no derivative at the points at which $sinx = 0$. (The function $f'(x)$suffers a discontinuity of the first kind). The solution to the last equation is the points$x_k = k\pi, k = 0; \pm1; \pm2; \pm3; \ldots$.

This means that the points $x_k = k\pi, k = 0; \pm1; \pm2; \pm3;$ ...are critical points of the function $f(x)$. The points $x_k = k\pi, k = 0; \pm1; \pm2; \pm3;$ ...are the maximum points of the given function according to Theorem 2, since when passing through these points the derivative of the given function changes sign from plus ("+") to minus ("-"), and at these points the given function is continuous. Indeed,

$$f'(x) = 1 \text{at} sinx < 0,$$
$$f'(x) = -1 \text{at} sinx > 0.$$

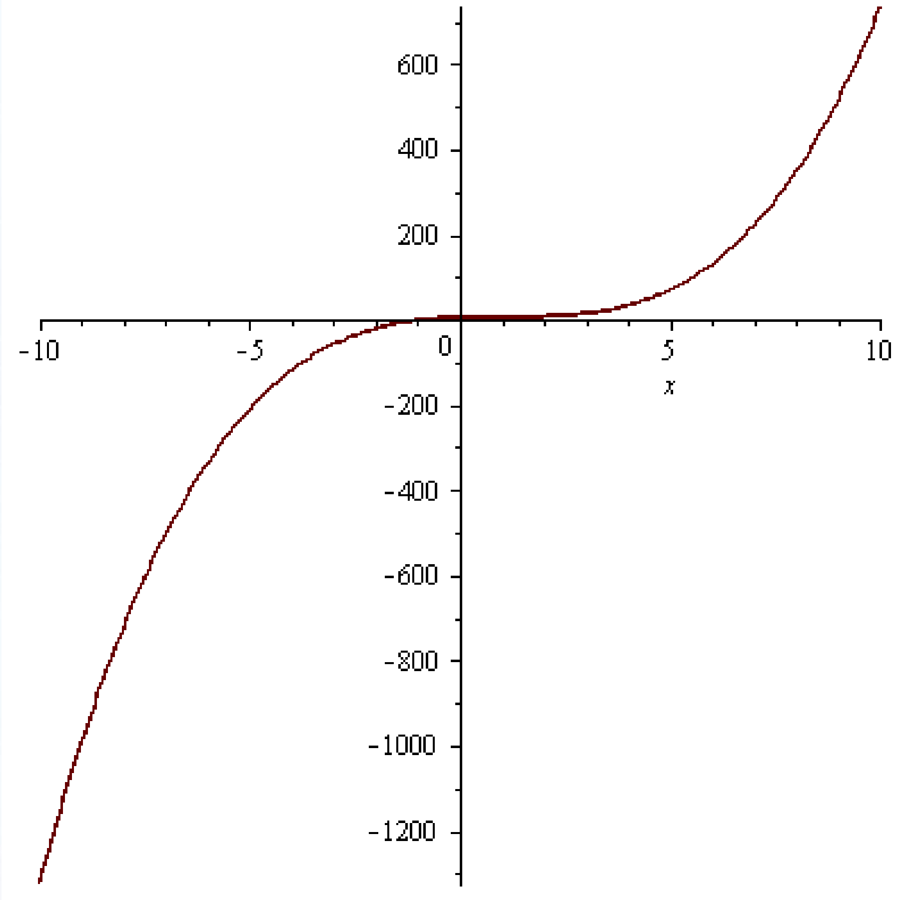
Similarly, the points $x_k = k\pi, k = 2n+1, n = 0; \pm1; \pm2; \pm3;$ ...are the minimum points of the given function according to Theorem 2, since when passing through these points, the derivative of the given function changes sign from minus ("-") to plus ("+"), and at these points the given function is continuous.



Example 6. Investigate the function for extremum

$$f(x) = x^3 - 3x^2 + 3x + 5.$$

>$plot(x^3 - 3 \cdot x^2 + 3 \cdot x + 5);$



Solution : The domain of the function $f(x)$is the entire number line. The derivative of this function is defined as:

$$f'(x) = 3x^2 - 6x + 3.$$

The solution to the equation $f'(x) = 0$, i.e.

$$3x^2 - 6x + 3 = 0$$

is a point $x = x_0 = 1$.The derivative of a given function exists on the entire number axis. Therefore, the point $x_0$is a critical point of the function $f(x)$. When passing through a critical point x_0, the derivative of a given function does not change sign. According to Theorem 2, at a critical point , x_0a given function does not have an extremum.

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